MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2016/2017

BSP2014 – INTRODUCTION TO APPLIED PROBABILITY AND STOCHASTIC PROCESSES

(All sections / Groups)

18 October 2016
2.30 p.m. – 4.30 p.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of **SIX (6)** printed pages inclusive of the cover page and formulae sheet.
2. Answer **ALL** four questions in the answer booklet provided.
3. Students are allowed to use authorized calculators by lecturer only.
4. Marks are shown at the end of each question.

Question 1 [25 marks]

(a) Given a random variable X has the following cumulative distribution function (CDF):

$$F(x)=\begin{cases}0 & , \quad x<-3\\ 1/8 & , \quad -3\le x<0\\ 3/8 & , \quad 0\le x<3\\ 5/8 & , \quad 3\le x<6\\ 1 & , \quad x\ge 6\end{cases}$$

(i) Construct the probability distribution for X. [4 marks]
(ii) Find $P(X \ge 0.5)$. [2 marks]
(iii) Determine $E(3X^3 + 2X^2 - 5)$. [9 marks]

(b) The random variable X is uniform on [20, 200],
(i) Find the probability density function for random variable X. [1 mark]
(ii) Find $P(100 < X \le 200)$. [3 marks]
(iii) Determine the moment generating function for X. [6 marks]

Question 2 [25 marks]

(a) Let X and Y be independent random variables with pdf

$$f(x)=\begin{cases}\frac{x}{2}, & 0\le x\le 2\\ 0, & \textit{otherwise}\end{cases} \qquad \text{and} \qquad f(y)=\begin{cases}\frac{2-y}{2}, & 0\le y\le 2\\ 0, & \textit{otherwise}\end{cases}$$

(i) Find E(XY). [7 marks]
(ii) Find Var (X – 2Y). [10 marks]

(b) On average, 5 customers arrive at one of the 7-eleven shop nearby MMU between 3:00am and 5:00 am.
(i) Find the probability that 2 customers arrive at this 7-Eleven shop between 8:30am and 9:00 am. [3 marks]
(ii) Find the probability that not more than 1 customer arrive at this 7-Eleven shop hourly. [5 marks]

Continued...

Question 3 [25 marks]

a) A random walk starts from the origin. The chances for the walker to move to the right are 4 times the chances he moves to the left.
   (i) Enumerate all possible paths to be at position 3 after 5 steps. [5 marks]
   (ii) Find the probability that the walker is at the – 2 after 6 steps. [6 marks]
   (ii) Find $E(X_{30})$ and $Var(X_{30})$. [4 marks]

b) The count of students skipping tutorial class is known to be a Poisson process of rate 0.5 skips per day. Let N(t) denote the number of students skipping tutorial class after t days from the first day of the trimester.
   (i) Determine the pmf for N(t). [2 marks]
   (ii) Find the probability of 3 students skipping tutorial class after 5 days. [4 marks]
   (iii) Find the probability of no one skipping the tutorial class in the first day. [4 marks]

Question 4 [25 marks]

Given the transition matrix, $P = \begin{bmatrix} 0.625 & 0.375 \\ 0.25 & 0.75 \end{bmatrix}$.

a) Draw the transition diagram. [3 marks]
b) Determine for every state whether it is an absorbing state. [4 marks]
c) Determine for every state whether it is transient or persistent. [12 marks]
d) Determine the stationary distribution. [6 marks]

End of Page

FORMULAE

A. PROBABILITY DISTRIBUTION

Bernoulli Probability Distribution

$$P(X = x) = p^x q^{1-x} \qquad \text{for } x = 0, 1$$

Binomial Probability Distribution

$$P(X = x) = {}^nC_x \, p^x q^{n-x} \qquad \text{for } x = 0, 1, \ldots, n$$

Poisson Probability Distribution

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!} \qquad \text{for } x = 0, 1, 2, \ldots$$

Geometric Probability Distribution

$$P(X = x) = pq^{x-1} \qquad \text{for } x = 1, 2, \ldots$$

Uniform Probability Distribution

$$f(x) = \frac{1}{b-a} \qquad \text{for } a < x < b$$

Exponential Probability Distribution

$$f(x) = \lambda e^{-\lambda x} \qquad \text{for } x > 0$$

Normal Probability Distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \qquad \text{for } -\infty < x < \infty$$

Gamma Probability Distribution

$$f(x) = \frac{x^{\alpha-1} e^{-x/\beta}}{\beta^\alpha \Gamma(\alpha)} \qquad \text{for } x \geq 0$$

where $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx$ for $\alpha \geq 0$

with $\Gamma(\alpha) = (\alpha - 1)\,\Gamma(\alpha - 1)$

with $\Gamma(n) = (n - 1)!$ when n is a positive integer.

Chi-Square Probability Distribution

$$f(x) = \frac{x^{\left(\frac{n}{2}-1\right)} e^{-x/2}}{2^{\frac{n}{2}}\Gamma\left(\frac{n}{2}\right)} \qquad \text{for } x \geq 0$$

B. MOMENT

k^{th} Moment about the origin of X:

$$\mu'_k = E[X^k]$$

k^{th} Moment about the mean of X:

$$\mu_k = E[X-\mu]^k$$

Moment Generating Function for X:

$$M_X(t) = E\left(e^{tx}\right)$$

Derivative of Moment Generating Function for X:

$$M_X^{(r)}(t) = E\left(X^r e^{tx}\right)$$

C. STOCHASTIC PROCESS

Simple Random Walk:

$$P(X_n = m) = \begin{pmatrix} n \\ \frac{n+m}{2} \end{pmatrix} p^{(\frac{n+m}{2})} q^{(\frac{n-m}{2})}$$ where m and n are both even or both odd.

$E(X_n) = n(p-q)$; $Var(X_n) = 4npq$

Gambler's Ruin Problem

$$P_a = \begin{cases} \dfrac{(\frac{q}{p})^a - (\frac{q}{p})^c}{1-(\frac{q}{p})^c}, & p \neq q \\ 1-\dfrac{a}{c}, & p = q \end{cases} \quad ; \quad P_b + P_a = 1$$

$$D_a = a(c-a) \quad \text{if} \quad p = q$$

$$D_a = \frac{1}{q-p}\left\{a - c\left[\frac{1-\left(\frac{q}{p}\right)^a}{1-\left(\frac{q}{p}\right)^c}\right]\right\} \quad \text{if} \quad p \neq q$$

D. POISSON PROCESS

Pmf for N(t) : $$P\{N(t)=n\}=\frac{e^{-\lambda t}(\lambda t)^n}{n!}$$
Pdf for T_n: $$f(t)=\lambda e^{-\lambda t}$$
Pdf for S_n: $$f(t)=\frac{\lambda e^{-\lambda t}(\lambda t)^{n-1}}{(n-1)!}$$